Glencoe Science

Mathematics Skill Activities

Student Edition

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Activity 1

NCTM Standard:

Number and Operations

Review

What you should know about . . .	Example
Fractions can be used to represent numbers greater than one.	$1\frac{1}{2} = \frac{3}{2} = 1.5 = 150\%$
Fractions, decimals and percents • can be used to represent the same number.	$\frac{1}{2} = 0.5 = 50\%$
• can be placed in order on a number line.	2.65 is less than 2.7
• can be used to show relationships such as ratios and rates.	"The survey showed that 30 percent of the cars tested averaged 10.8 km/L."
Negative and positive integers How do you determine their values?	–35 is less than –3, and $-72 - (-41) = -31$.
Scientific notation How is it used to express large numbers?	4.3 billion can be written as 4,300,000,000, or as 4.3×10^9.

Example

The U.S. Geological Survey keeps track of many species of wildlife. **Table 1** shows information gathered about North American birds in five states. The percentage of species in a state with population increases was determined using population data from 1980–1999. In which state did approximately $\frac{1}{2}$ of the species experience population increases?

Table 1

State	Species with population increases (%)
Colorado	59
Minnesota	50
New York	45
Ohio	56
Texas	40

Source: U.S. Geological Survey

Solution To answer this question, first express the percentages as fractions. Then decide which fraction is closest to $\frac{1}{2}$. A percent is a fraction in which the denominator is 100. The fractions for the five states are Colorado: $59\% = \frac{59}{100}$, Minnesota: $50\% = \frac{50}{100}$, New York: $45\% = \frac{45}{100}$, Ohio: $56\% = \frac{56}{100}$, and Texas: $40\% = \frac{40}{100}$. Find the fraction that is closest to $\frac{1}{2}$. Because $\frac{1}{2} = \frac{50}{100}$, Minnesota is the correct answer.

Activity 1 (continued)

Practice

There are 9 planets in our solar system. **Table 2** shows each planet's average distance from the Sun in astronomical units (AU). One AU is equal to about 150 million km.

Table 2

Planet	Average distance from the Sun (AU)
Mercury	0.39
Venus	0.72
Earth	1.00
Mars	1.52
Jupiter	5.20
Saturn	9.54
Uranus	19.18
Neptune	30.06
Pluto	39.53

Source: NASA

1. Which planet has a distance from the Sun that is about 150% of Earth's distance from the Sun?
a. Mercury
b. Jupiter
c. Mars
d. Venus

2. How far is Earth from the Sun?
a. 15×10^9 km
b. 1.5×10^7 km
c. 15×10^5 km
d. 1.5×10^8 km

3. Approximately how many times farther from the Sun is Neptune than Jupiter?
a. 3
b. 6
c. 10
d. 12

4. In a chemistry experiment, 8 out of 10 groups got the same results. How is this ratio expressed as a percent?
a. 20%
b. 40%
c. 50%
d. 80%

5. Neon gas has a boiling point of −246°C. Xenon gas has a boiling point of −108 °C. How much warmer does it have to be for xenon gas to boil than for neon gas?
a. 138°C
b. 354°C
c. 216°C
d. 108°C

Name Date Class

Activity 2

NCTM Standard:

Number and Operations

Review

What you should know about . . .	Example
Multiplying fractions Multiply numerators, then multiply denominators.	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
Dividing fractions Invert the divisor, then multiply.	$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1\frac{1}{2}$
Adding and subtracting fractions Find the common denominator, then add or subtract the numerator.	$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$ $\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$
Use **associative, commutative,** and **distributive properties** to make it easier to work with fractions.	$4\left(\frac{1}{2} + \frac{2}{3}\right) = \left(4 \times \frac{1}{2}\right) + \left(4 \times \frac{2}{3}\right) =$ $\frac{4}{2} + \frac{8}{3} = 2 + 2\frac{2}{3} = 4\frac{2}{3}$
Inverse relationships • Multiplication is the inverse of division. • Addition is the inverse of subtraction. • Squaring is the inverse of finding the square root.	$\sqrt{17}$ is less than $\sqrt{50}$ because $4^2 = 16$ and $7^2 = 49$, and 16 is less than 49.

Example

Dane and Beth need salt water for a chemistry experiment. The instructions ask them to mix $1\frac{1}{2}$ cups of water with $\frac{1}{4}$ cup of salt. They discovered just before they started that they only have $\frac{1}{2}$ as much salt as they need. How much salt do they have and how much water should they mix with it in order to get salt water with the concentration they need?

Solution First, find out how much. salt they actually have.

$$\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$$

They have $\frac{1}{8}$ cup of salt. If they have only $\frac{1}{2}$ the amount of salt they need, they should mix it with $\frac{1}{2}$ the amount of water that the instructions ask for.

$$1\frac{1}{2} \text{ cups of water} = \frac{3}{2} \text{ cups of water}$$

$$\frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

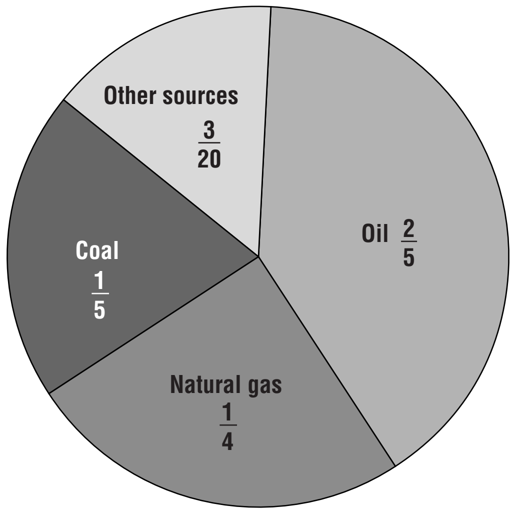
Therefore, they should mix $\frac{1}{8}$ cup of salt with $\frac{3}{4}$ cup of water in order to get salt water with the concentration that they need.

Activity 2 (continued)

Practice

Most of our energy in the United States comes from fossil fuels. The circle graph in **Figure 1** shows which fractions of the total energy in the United States come from coal, oil, natural gas, and other sources.

Figure 1 **Sources of U.S. Energy**



Source: U.S. Department of Energy

1. How much more of the total amount of energy is produced from oil than from natural gas?
   - **a.** $\frac{3}{20}$
   - **b.** $\frac{1}{4}$
   - **c.** $\frac{1}{5}$
   - **d.** $\frac{9}{20}$

2. How many times more energy is generated by coal than by other sources (not oil or natural gas)?
   - **a.** 3
   - **b.** $2\frac{1}{2}$
   - **c.** 2
   - **d.** $1\frac{1}{3}$

3. What fraction of energy is produced by natural gas and coal together?
   - **a.** $\frac{2}{5}$
   - **b.** $\frac{1}{4}$
   - **c.** $\frac{9}{20}$
   - **d.** $\frac{1}{20}$

4. To find the probability of two events happening, multiply the probabilities of the individual events. If the chance that a couple's first child will be a girl is $\frac{1}{2}$ and the chance that their second child will be a girl is also $\frac{1}{2}$, what is the chance that both of their children will be girls?
   - **a.** $\frac{1}{8}$
   - **b.** $\frac{1}{4}$
   - **c.** $\frac{1}{6}$
   - **d.** 1

5. Four square areas of land each have 50 deer living on them. The sides of area A are the square root of 65 km, the sides of area B are 7 km, the sides of area C are 6.3 km, and the sides of area D are the square root of 40 km. Which area has the most space per deer?
   - **a.** area A
   - **b.** area B
   - **c.** area C
   - **d.** area D

Activity 3

NCTM Standard:

Number and Operations

Review

What you should know about . . .	Example
Estimating Know when to estimate and when an exact amount is needed.	You can **estimate** how much money you will need to take to the grocery store. However, you will need to calculate the **exact amount** of your purchases to know how much money you will need at the checkout counter.
Proportions How can you use proportions to solve problems?	Which is a better value? 100g of laundry detergent for \$3.59 or 500g of laundry detergent for \$5.29? 3.59 ÷ 100 = \$0.0359/g 5.29 ÷ 100 = \$0.01058/g

Example

Coal forms over long, long periods of time from plants that die and decompose beneath soil and often rock. The plant matter first turns into peat before it becomes coal. Geologists estimate that in Kentucky, it would have taken a layer of peat measuring about 3 m deep to produce a layer of coal measuring about 0.3 m thick. How many meters deep would a layer of peat have to have been to produce a coal seam measuring 15 m deep?

Solution Solve this problem by writing a proportion. A coal seam measuring 0.3 m thick is made over time from a 3-m deep layer of peat. Find out how much peat is needed to make 15 m of coal by writing the proportion $\frac{3}{0.3} = \frac{x}{15}$. To solve for x, multiply 3 × 15 and divide by 0.3. It would take 150 m of peat to produce 15 m of coal.

Practice

1. What is the average high temperature for the summer in Kotzebue?
   a. 38.78°C
   b. 19.39°C
   c. 13.89°C
   d. 12.93°C

Table 1

Average High Temperatures in Kotzebue, Alaska

Month	Average High Temperature (°C)
June	9.83
July	15.06
August	13.89

Source: National Climatic Data Center

2. Estimate about how many degrees warmer it is in July than in June in Kotzebue.
   a. 4°C
   b. 5°C
   c. 6°C
   d. 7°C

Activity 3 (continued)

3. A farmer normally plants 40,000 corn plants on a 5-hectare field. This year she is letting 2 hectares of the field rest and will only plant 3 hectares. How many plants does she need for the 3 hectares?

a. 16,000 **c.** 120,000
b. 24,000 **d.** 200,000

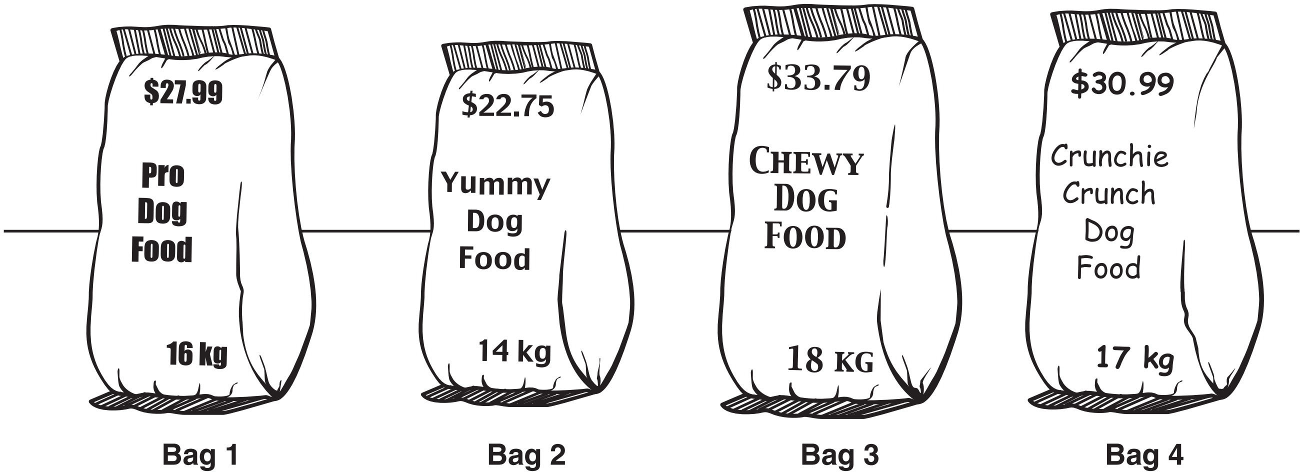
4. In the spring of 1995, the number of California sea otters spotted along the California coast was the highest since the fall of 1982. Approximately 2,400 otters were seen. But between 1995 and 1998, this number started falling, reaching a low near 1,900 by the fall of 1998. If this trend continued, about how many California sea otters would be counted in California in 2004?

a. 3,000 **c.** 1,500
b. 2,900 **d.** 900

5. Kari has a large dog that eats a lot of food. She always buys the dog food that is the best value at the pet store. Which bag of food is the best value?

a. Bag 1 **c.** Bag 3
b. Bag 2 **d.** Bag 4

Figure 1

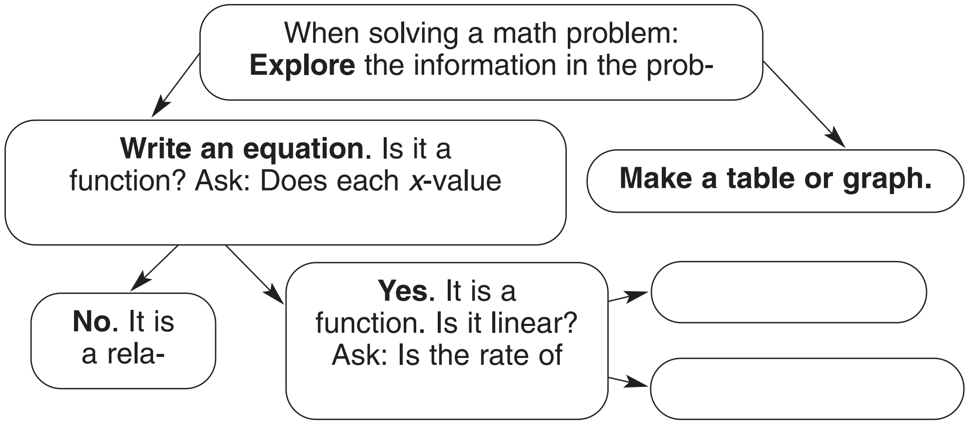


Activity 4

NCTM Standard:

Algebra

Review



Example

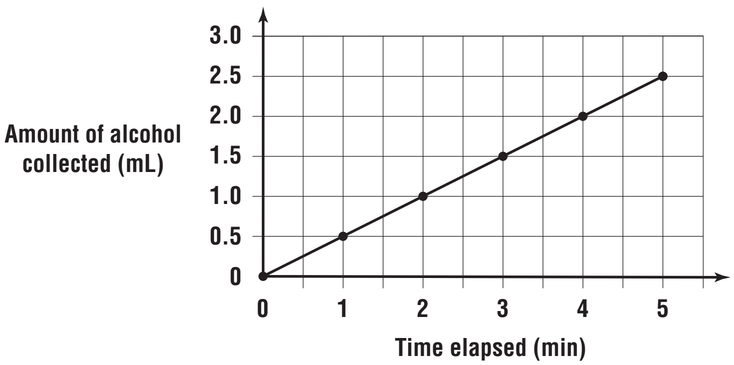
Sally and Arun are distilling a solution of water and isopropyl alcohol to separate the two liquids. Alcohol boils off faster than water, so it will be the first liquid collected. They have set up the experiment so they can measure the amount of alcohol they collect each minute. They begin collecting data when the mixture starts boiling. At 0 min, no alcohol has been distilled. At 1 min, they collect 0.5 mL. At 2 min, they collect 1.0 mL. They forget to check it at 3 min, but at 4 min, they collect 2.0 mL. At 5 min, they collect 2.5 mL. How many milliliters were in the collection at 3 min?

Solution First, organize the data into a table. This will make them easier to evaluate.

Table 1

Time elapsed (min)	Amount collected (ml)
0	0.0
1	0.5
2	1.0
3	?
4	2.0
5	2.5

Next, graph the data.



Then, find the amount collected at 3 min on the graph. At 3 min, Sally and Arun had collected 1.5 mL.

Activity 4 (continued)

Practice

A bit (short for binary digit) is the smallest unit of information a computer can hold. A computer uses groupings of 8 bits, called bytes, to process information. A single byte is the amount of memory needed to represent one letter, number, or symbol in the computer. The amount of information in a computer is measured in multiples of bytes. Look at **Table 2.** It shows the names for large numbers of bytes and the multipliers used to obtain these figures. A kilobyte is 1,024 bytes (2^{10}) or about 1,000 bytes. A megabyte is 1,048,576 bytes or about a million bytes.

Table 2

Term	Abbreviation	Size
Kilobyte	K	2^{10} = 1,024
Megabyte	M	2^{20} = 1,048,576
Gigabyte	G	2^{30} = 1,073,741,824
Terabyte	T	2^{40} = 1,099,511,627,776
Petabyte	P	2^{50} = 1,125,899,906,842,624
Exabyte	E	2^{60} = 1,152,921,504,606,846,976

1. About how many bytes are there in 3 G?
A. 3 million **B.** 300,000 **C.** 3 billion **D.** 300 million

2. If a data compact disc holds about 650 M of information, about how many CDs would you need to use in order to download 2 G of information from your computer?
A. 1 **B.** 2 **C.** 3 **D.** 4

Atmospheric pressure decreases as altitude increases because the number of air molecules decreases. At sea level, the amount of atmospheric pressure is about 1 kg/cm^2. At an altitude of about 5.5 km above sea level, atmospheric pressure is half of what it is at sea level. Assume that between 0 km and 5.5 km, the rate of decrease in atmospheric pressure is constant.

3. If the atmospheric pressure is 1 kg/cm^2 at sea level, what would the pressure be at 22 km?
A. 0.1 kg/cm^2 **B.** 1.0 kg/cm^2 **C.** 1.1 kg/cm^2 **D.** 2.3 kg/cm^2

4. At what altitude would the air pressure be about $\frac{3}{4}$ of what it was at sea level?
A. 2.75 km **B.** 27.5 km **C.** 16.5 km **D.** 1.65 km

Activity 5

NCTM Standard:

Algebra

Review

What you should know about . . .	Example
Variables	Letters or symbols that represent values
Equations	Show relationships Example: $y = mx + b$ describes a line, where y and x represent the points on the line, m gives the slope of the line, and b tells where the y-intercept of the line lies.
Order of operations	**1.** Do the operations in parentheses or brackets first. **2.** Solve powers or exponents before other operations. **3.** Working from left to right, multiply and divide. **4.** Working from left to right, add and subtract.

Example

The division of cells in an animal's body is called mitosis. In mitosis, each cell divides to form two new cells. In the equation $y = 2^x$, y is the number of cells you will have if one cell goes through mitosis x number of times. How many cells would be created from one original cell if the cell went through mitosis 4 times?

Solution Follow the order of operations to solve the equation $y = 2^x$ for $x = 4$. The equation becomes

$$y = 2^4$$

$$y = 2 \times 2 \times 2 \times 2 = 16.$$

There would be 16 cells made from the original cell if it divided 4 times.

Activity 5 (continued)

Practice

In the equation $d = vt$, d = distance, v = velocity, and t = time.

1. How far does a person on a bicycle traveling at 20 km/h travel in 30 min?

A. 10 km **C.** 30 km
B. 20 km **D.** 40 km

2. Bottlenose dolphins can swim 30 km/h if they work hard. How long would a dolphin have to swim at this speed to travel 22.5 km?

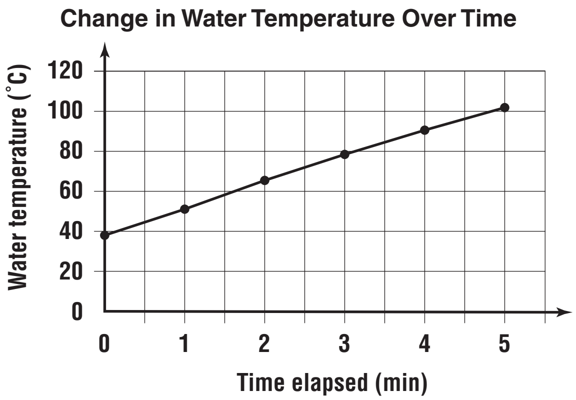
A. 2 h **C.** 30 min
B. 1 h **D.** 45 min

3. It is common in the United States to report temperatures using Fahrenheit degrees. However, much of the world uses the Celsius temperature scale. To change a Fahrenheit temperature to a Celsius temperature, use the equation $(F - 32)\frac{5}{9} = C$, where F is the Fahrenheit temperature and C is the Celsius temperature. How many degrees Celsius is 77°F?

A. 20°C **C.** 30°C
B. 25°C **D.** 35°C

4. Students in Mrs. McGovern's chemistry class applied constant heat to water and measured its temperature every minute to see whether its temperature would increase at a constant rate. Based on the graph in **Figure 1**, what was the approximate starting temperature of the water?

Figure 1



A. 0°C **C.** 39°C
B. 23°C **D.** 103°C

Activity 6

NCTM Standard:

Algebra

Review

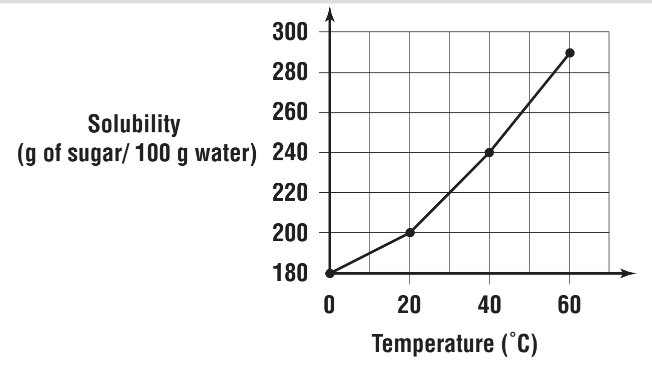
What you should know about . . .	Example
Making sense of data	Ask these questions: • Do the data represent a linear relationship or a nonlinear one? • Do they show a trend from which you can predict what will happen next?
Making a model of data	Making a graph can help you make sense of data.
Are data reliable?	Compare graphs to find out whether data are reliable.

Example

Standard granulated sugar can be dissolved in water. The amount of sugar that will dissolve in 100 g of water changes as the temperature of the water changes. At 0°C, about 180 g of sugar will dissolve. At 20°C, about 200 g of sugar will dissolve. At 40°C, about 240 g of sugar will dissolve. At 60°C, about 290 g of sugar will dissolve. About how many g of sugar will dissolve in 100 g of water at 30°C?

Solution To answer the question, first graph the data. Place the temperature data on the *x*-axis and the solubility data on the *y*-axis. Since the lowest solubility number is 180, it may be helpful to start the *y*-axis at 180.

Figure 1



Next, find how many grams of sugar will dissolve in 100 g of water at 30°C on the graph. According to the graph, about 220 g of sugar will dissolve in 100 g of water at 30°C.

Name Date Class

Activity 6 (continued)

Practice

1. **Table 1** shows the number of bacteria in an actively dividing population each half hour. Study **Table 1**, looking for a pattern. How many bacteria would there be after 5 hours?
   a. 256 **b.** 512 **c.** 1,024 **d.** 2,048

Table 1

Time elapsed (h)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Number of bacteria	1	2	4	8	16	32	64	128	256

For a physics experiment, Marc's class pushed a toy car across the floor and measured how far it had moved every 2 s. Then, they used the equation velocity = distance ÷ time $\left(v = \frac{d}{t}\right)$ to calculate the car's velocity.

Figure 2

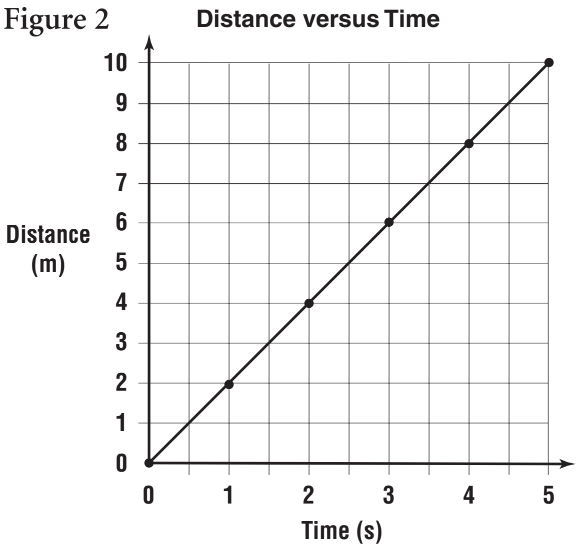
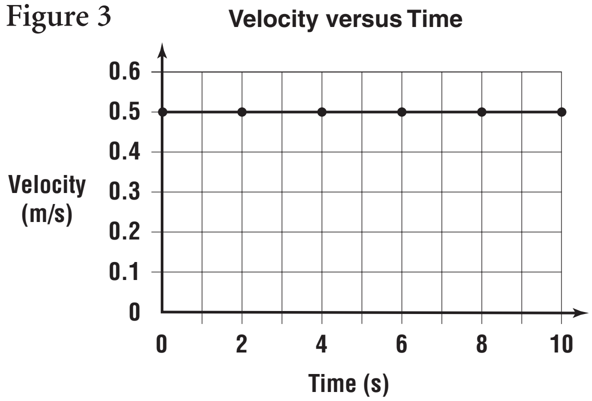


Figure 3



2. Why does the y-value of the velocity vs. time graph stay constant?
   a. The car is increasing its velocity.
   b. The car is slowing down.
   c. The car is not moving.
   d. The car is moving at a constant velocity.

3. What does the y-value at 6 s represent on the distance vs. time graph?
   a. the distance the car had traveled at 6 s
   b. the distance the car travels per second
   c. the velocity the car had at 6 s
   d. the velocity of the car per second

4. Explain why the two graphs look different even though the data are the same.

Activity 7

NCTM Standard:

Geometry

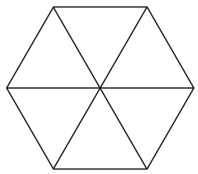
Review

What you should know about . . .	Example
Two-dimensional shapes	Square, rhombus, rectangle, circle, triangle, ellipse, and polygon
Three-dimensional shapes	Sphere, cube, cone, cylinder, and pyramid
Geometric properties	Each shape has unique properties that define it. Example: All sides of a square are equal. Example: The diagonals of a parallelogram bisect one another.
Congruency	Shapes and angles that fit exactly one on top of another are congruent.
Similarity	Shapes that have the same proportions are similar. For example, they may have congruent angles, but not congruent sides.

Example

The crystal structure of a hydrogen atom is hexagonal like the shape in the figure. If a hexagon is bisected three times, six shapes are formed. Are these six shapes similar, congruent, or neither?

Figure 1



Solution Look at the picture of the hexagon above. The six shapes formed by bisecting the hexagon are all triangles. Because the six sides of a hexagon are equal, the triangles all have bases of equal length. They also have sides of equal length. The triangles have the same shape and the same size, making them congruent.

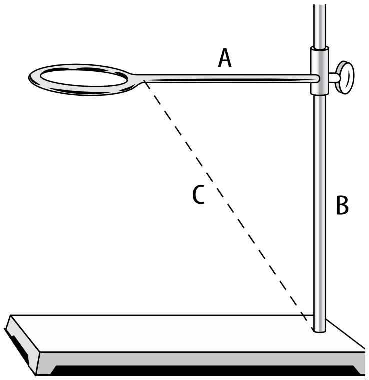
Name Date Class

Activity 7 (continued)

Practice

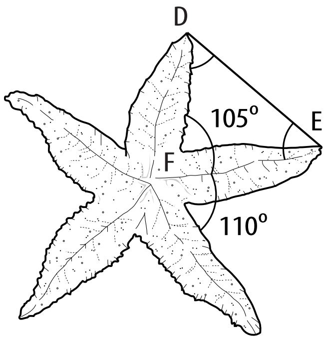
1. Assume that the ring stand in **Figure 2** forms a right angle. How long is C if A = 32 cm and B = 46 cm?
   a. 3140.0 cm **c.** 78.0 cm
   b. 56.0 cm **d.** 14.0 cm

Figure 2



2. What is the measurement of $\angle E$, assuming that the vein $\overline{EF}$ in the arm of the starfish is a straight line?
   a. 145° **c.** 70°
   b. 60° **d.** 75°

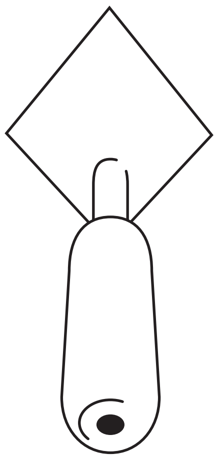
Figure 3



3. Archaeologists dig to find fossils and artifacts from ancient cultures. The head of a trowel, one of the tools that they use, is shaped like a diamond with four congruent sides. Of the following choices, which is the most accurate name for this shape?
   a. cube **c.** rectangle
   b. square **d.** rhombus

4. Erin is counting the number of different trees in a rectangular area that measures 4 m × 5 m. Next, she needs to count the trees in an area twice as big. What would the dimensions of the bigger area be?
   a. 8 m × 5 m **c.** 8 m × 10 m
   b. 2 m × 10 m **d.** 10 m × 20 m

Figure 4



Name Date Class

Activity 8

NCTM Standard:

Geometry

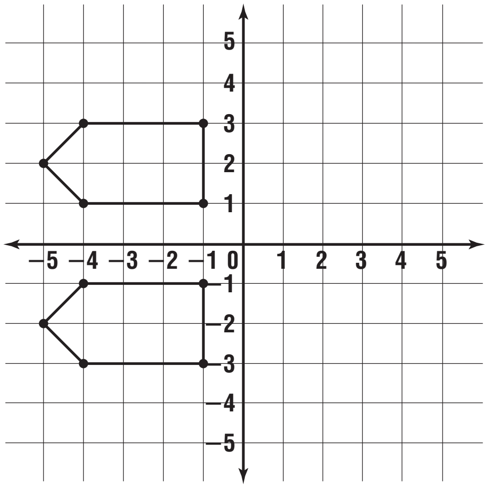
Review

Geometric shapes can be plotted on a coordinate grid to study them in greater detail. For instance, the slopes of the sides of a triangle can be calculated from the coordinates of the vertices. Also, symmetry can be studied by reflecting or rotating a shape about a line or a point and then examining the resulting shape. In the same way, a shape can be reduced or enlarged by dilation to form a second shape similar to the first.

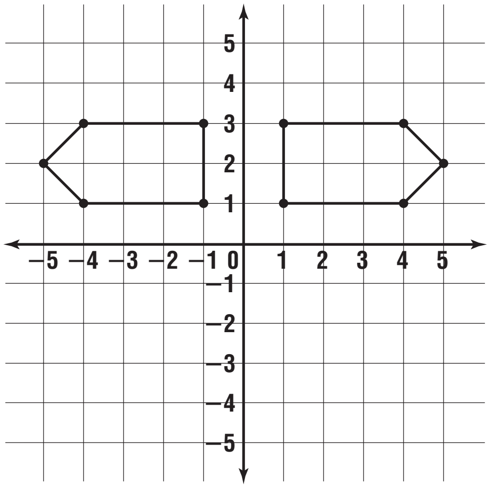
Example

The wings of an airplane are reflections of each other over the body of the airplane. If the *y*-axis represents the body of the airplane, which graph shows the orientation of its wings?

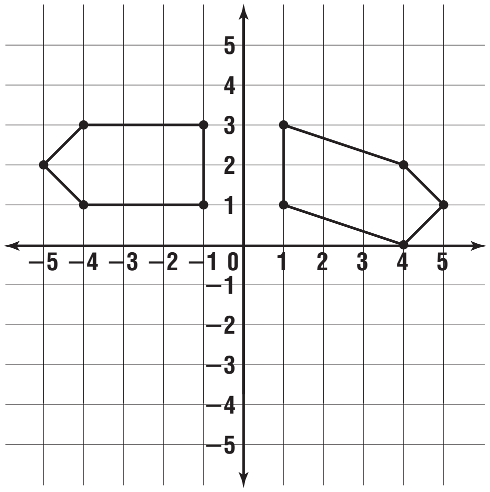
a.



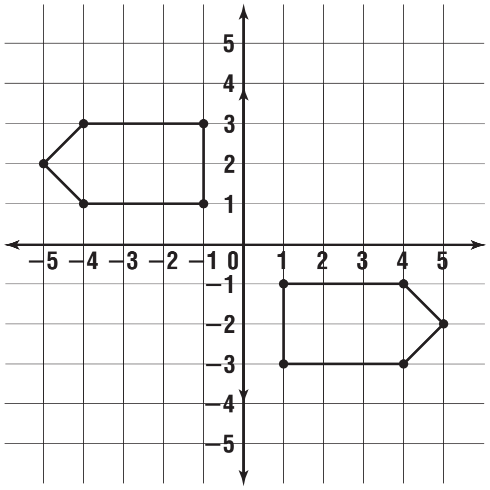
c.



b.



d.



Solution To find a reflection over the *y*-axis, look for a mirror image on the other side of the *y*-axis, the same distance from the axis. Graph **c** is a mirror image over the *y*-axis. Therefore, **c** is the correct answer.

Activity 8 (continued)

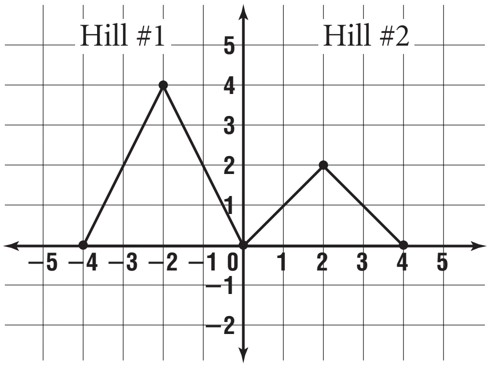
Practice

Figure 1 represents the first and second hills of a roller coaster.

1. What is the slope going up the first hill?
   a. $-\frac{1}{2}$ c. 2
   b. $\frac{1}{2}$ d. -2

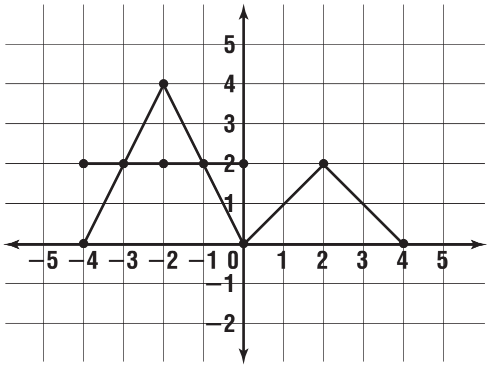
2. What is the slope coming down the second hill?
   a. -1 c. $-\frac{1}{4}$
   b. 4 d. -4

Figure 1

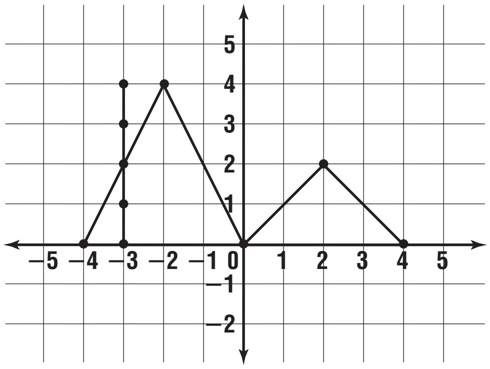


3. Which of these graphs shows the line of symmetry of the first hill?

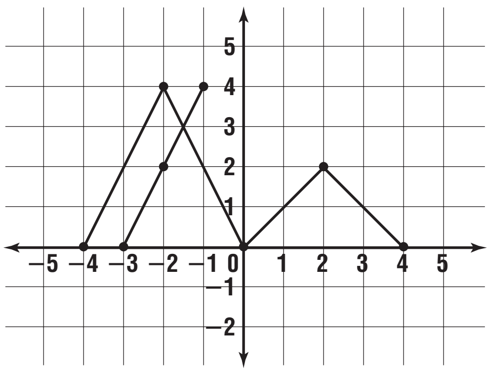
a.



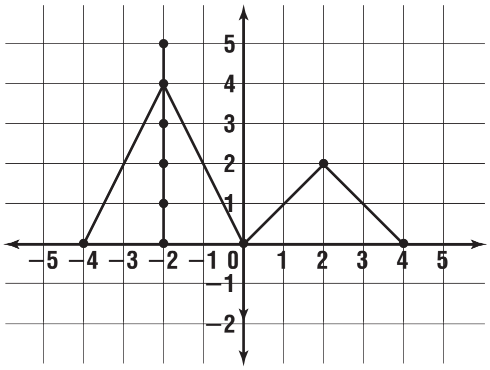
c.



b.



d.



Name Date Class

Activity 9

NCTM Standard:

Geometry

Review

Using physical models can make it easier to examine the properties of geometric shapes.

- Turn and flip the cut-out of a triangle to understand rotations and translations.
- Use three-dimensional models to understand two-dimensional representations of three-dimensional objects.
- Use networks to solve real-world problems. A network maps the paths that a person, vehicle, or task might take.

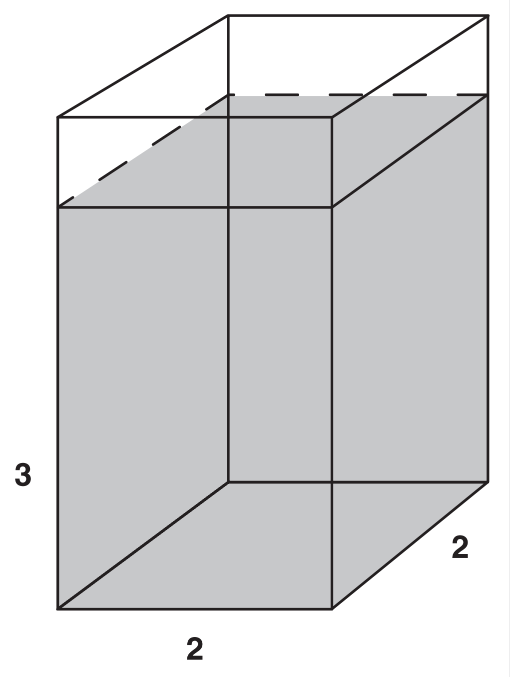
  Example: Trace all of the steps you take during science lab one afternoon for later analysis. Once the routes are on paper, you can examine whether you worked as efficiently as possible or whether you could have cut out some steps.

Example

Picture a square-shaped rain gauge that measures 2 cm × 2 cm on the bottom. If the rain in it extends 3 cm up the side, how much rain is in the rain gauge?

Solution Make a sketch of the rain gauge as shown in **Figure 1**. If the rain gauge has a square bottom, then it is in the shape of a rectangular prism. The amount of rain in the rain gauge is the volume of rain. To find the volume of a rectangular prism, multiply its width by its depth by its height. In this problem, the width and the depth both measure 2 cm and the height of the water is 3 cm. Because $2 \times 2 \times 3 = 12$, there are 12 cm^3 in the rain gauge. Cubic centimeters are equal to milliliters, so there are 12 mL of rain in the rain gauge.

Figure 1



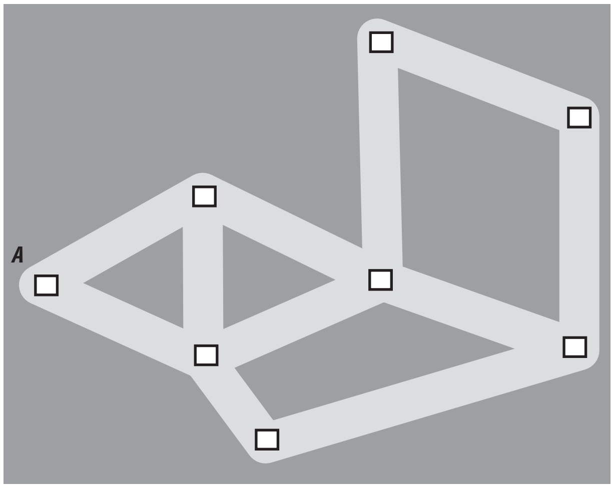
Practice

Scientists use models to show how electrons are arranged in molecules. For example, the bonds between hydrogen and oxygen atoms in water molecules joined together in ice form tetrahedrons. A tetrahedron is a solid with 4 vertices and 4 faces, with each face being an equilateral triangle.

Activity 9 (continued)

1. Draw four equilateral triangles of the same size on paper and cut them out. Put them together so that the top vertices of 3 of the triangles meet at a point, then tape the sides together. Align the fourth triangle so that its edges meet the bottom edges of the other triangles, then tape the edges. Draw the tetrahedron you have made in the space provided and describe its shape.

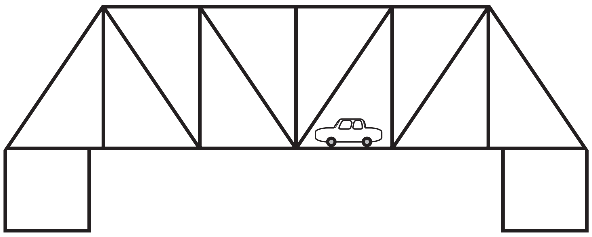
Figure 2



2. A beekeeper examines the honeycombs from her beehives once a day. **Figure 2** shows the location of the beehives. Starting at point A, draw the most efficient path for the beekeeper to take between beehives so that she does not go back past beehives she has already checked.

Triangles often are found in bridges. Look at the diagram of a bridge in **Figure 3**. The triangles are congruent.

Figure 3



3. If the bridge is 192 m across, how long is the base of each triangle?
   a. 25 m
   b. 32 m
   c. 55 m
   d. 64 m

4. If the height of the triangles in the bridge is 75 m each, what is the area of one of the triangles?
   a. 800 m^2
   b. 2,800 m^2
   c. 2,400 m^2
   d. 1,200 m^2

Activity **10**

NCTM Standard:

Measurement

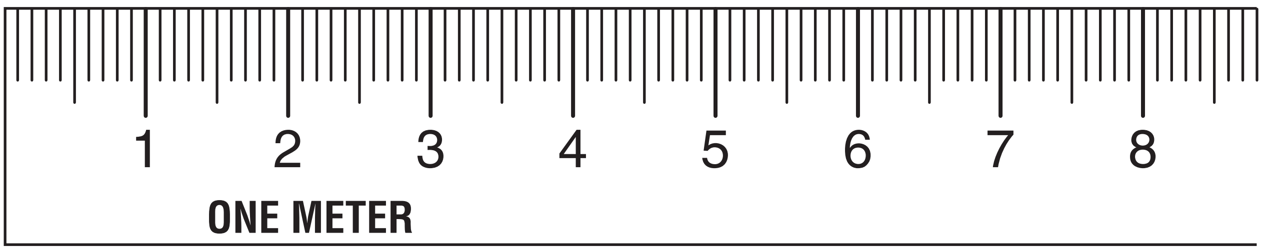
Review

Measuring items precisely is crucial in the science laboratory. Therefore, you should be familiar with various units of measurement, particularly the International System of Units (SI). Often, you will need to convert from one unit to another. For example, you may need to find how many millimeters in length something measures. If your ruler only shows inches and centimeters, it is necessary to know that 1 cm equals 10 mm, and that $\frac{1}{4}$ cm is 2.5 mm. You also should be familiar with the uses for particular units. You wouldn't measure an inchworm in kilometers, and you certainly wouldn't measure temperature in liters. Understanding SI units ahead of time will make lab experiments go much more smoothly.

Example

The directions for a science experiment ask you to cut a 5-m long string into 50-cm lengths, but you only have a meterstick. How would you use the meterstick to cut the string accurately?

Solution There are 100 cm in 1 m. You easily can change meters to centimeters by moving the decimal point two spaces to the left. For example, 50 cm is equal to 0.5 m. Thus, each 50-cm length of string that you need to cut would be half as long as the meterstick. So, you could cut 5 pieces of string from the original piece using the meterstick, and then fold each resulting piece of string in half to know where to cut. This will make 10 pieces of string 50-cm long.



Practice

1. One variety of giant African snails, called *Achatina achatina,* is perhaps the world's largest land snail. It can grow to be around 20 cm long. How many millimeters is this?
   a. 2,300 mm
   b. 200 mm
   c. 2 mm
   d. 0.2 mm

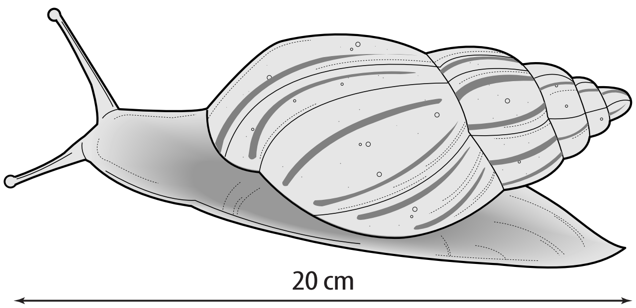
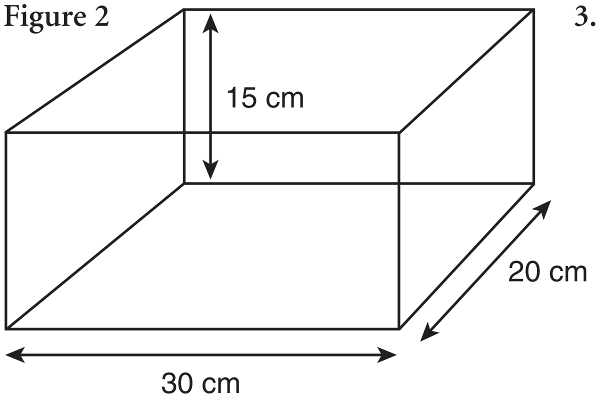


Figure 1

Activity 10 (continued)

2. Of the following, which is the most appropriate unit to measure the weight of an elephant?
   - **a.** kilograms
   - **b.** grams
   - **c.** milligrams
   - **d.** decagrams

Figure 2



3. If the dimensions of a rectangular aquarium are 30 cm wide × 20 cm deep × 15 cm high, what is its volume?
   - **a.** 9,000 mm
   - **b.** 9,000 cm
   - **c.** 9,000 cm^2
   - **d.** 9,000 cm^3

4. The dinosaur *Plateosaurus* lived about 210 to 195 million years ago and was the first animal capable of eating leaves high up in a tree. Scientists believe that it measured 9 m long and 3 m to 4 m high and that it weighed about 700 kg. *Plateosaurus* probably could stand on its rear legs while pulling down branches with its clawed hands. A person 6 feet tall is about 183 cm tall. How long was *Plateosaurus* in feet?
   - **a.** 0.295 ft
   - **b.** 1.22 ft
   - **c.** 29.5 ft
   - **d.** 122 ft

Figure 3

5. A scientist wants to count a sample of insects and worms living in the soil of a rain forest. Which size area will provide a good sample and allow the scientist to get an accurate count efficiently?
   - **a.** 1 mm^2
   - **b.** 1 cm^2
   - **c.** 1 k^2
   - **d.** 1 m^2

6. Of the following, which unit would you most likely use to measure a liquid for a science experiment?
   - **a.** centimeters
   - **b.** grams
   - **c.** milliliters
   - **d.** barrels

Name Date Class

Activity **11**

NCTM Standard:

Measurement

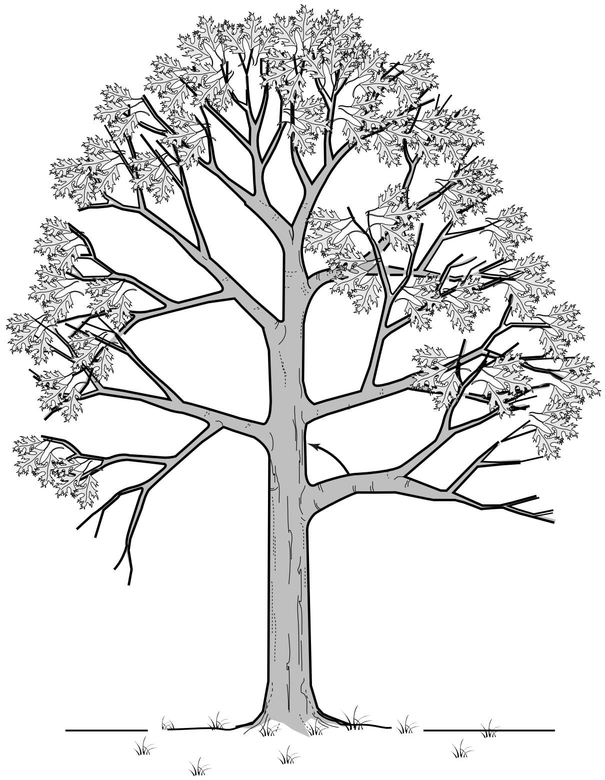
Review

How can you know whether the measurements you have for an item make sense? If the angle of a triangle is given as 185°, knowing that all of the angles of a triangle always add up to 180° will tell you that the given measurement is incorrect. Likewise, knowing basic formulas of measurement can eliminate unnecessary steps in solving a problem. For example, it is much easier to measure a circle's radius than its circumference. Knowing that a circle's circumference is always equal to $2\pi r$, where r is the radius of the circle, will save you a lot of work.

Example

Scientists classify and identify trees according to common characteristics among a species, such as leaf arrangement, leaf shape, bark appearance, and the general shape of the tree. What is a reasonable estimate for the angle made by one branch (marked in **Figure 1**) and the trunk of this scarlet oak tree?

Figure 1



Solution Look at the angle. Compare it to common angle sizes you know. A 90° angle will look like one corner of a square. Is this angle less than or greater than 90°? It is less than 90°, by about a third. A good estimate is 60°.

Practice

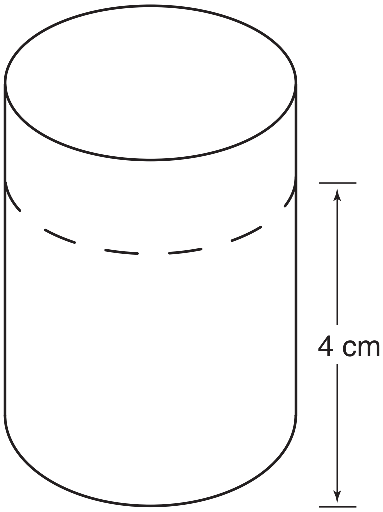
1. A science experiment asks you to measure the pH level of milk. You and your lab partner determine the pH of your sample of milk to be 6.1. The standard or accepted pH level of milk is 6.5. Using the formula absolute error = observed value − accepted value, what is the absolute error of this measurement?

   a. ± 0.4 **c.** ± 1.0
   b. ± 0.5 **d.** ± 4.0

Activity 11 (continued)

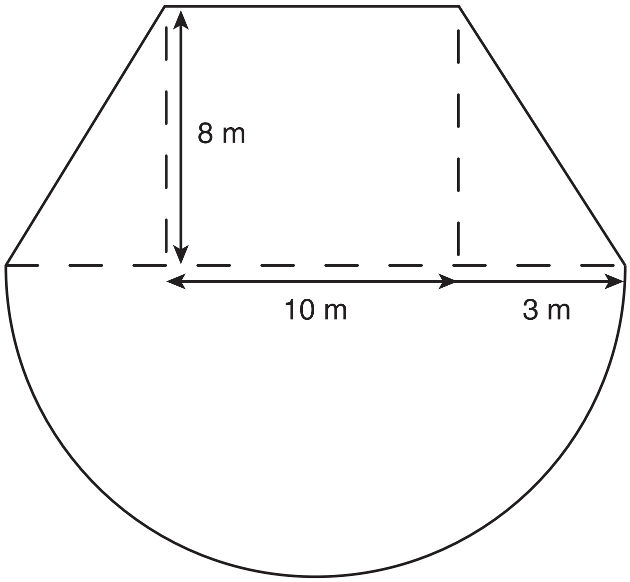
2. For a chemistry experiment, you collect water in a large beaker. The beaker does not have measurements to tell the volume of water collected. The level of water rises 4 cm from the bottom of the cylinder and the radius of the cylinder measures 3 cm. How would you find the volume of the water in the beaker?

Figure 2



3. At a nature reserve, the staff is working on improving some of the trails. Staff members want to pour new gravel at the site of a fallen tree. The space is shown in **Figure 3**. What is the area of the site?

Figure 3



a. 305.0 m^2
b. 204.5 m^2
c. 192.5 m^2
d. 261.0 m^2

4. If the nature reserve staff decided to pour gravel on only $\frac{2}{5}$ of the above area, what would be the diameter of the half-circle of the new space?

a. 4.0 m
b. 4.5 m
c. 3.2 m
d. 6.4 m

Activity 12

NCTM Standard:

Data Analysis and Probability

Review

How will you represent your data after conducting a lab acivity?
For example:

Lab:

Four students count the number of plants in each square meter of ground.

Representation:

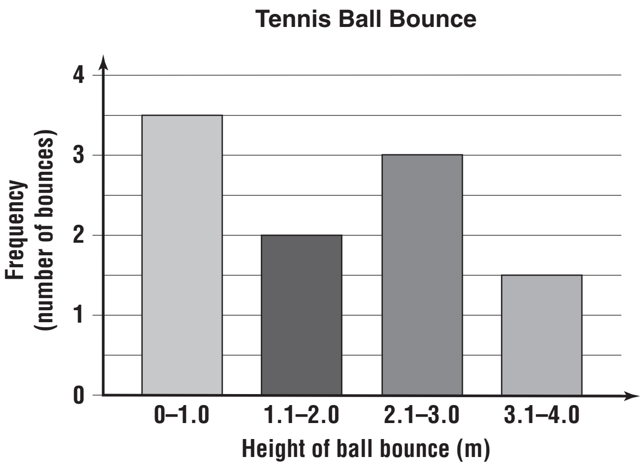
- They use an **absolute-frequency** graph to show the actual number of plants in each square.
- They use a **relative-frequency** graph to show the percent of the total number of plants for each square.

Other types of graphs, sucha as histograms, scatter plots, circle graphs, and box-and-whisker plots, can be use to represent data as well.

Example

Sergio and Chloe measured how high a tennis ball bounced when it was dropped from a height of 1.5 m. They did this 10 times. The data they collected are shown in **Figure 1**.

Figure 1



How many times did the tennis ball bounce 3.0 m or higher?

Solution Read the question carefully. It asks how many times the ball bounced 3.0 m or higher. You need to find out how many times it bounced 3.0 m plus how many times it bounced higher than that. According to the histogram, it bounced 3.0 m 3 times. It also bounced 3.5 m 1 time. The answer is 4 times.

Practice

Fruit flies often are used in genetic experiments. They reproduce quickly and are easy to study. Fruit flies normally have red eyes. But sometimes a mutation causes a fruit fly to have white eyes. Two students counted 120 flies from a cross between a red-eyed male and a red-eyed female fruit fly. Even though the male had red eyes, he carried the gene for the white-eyed mutation. The data they collected are shown in **Table 1**.

Table 1

Type of fruit fly	Number of flies
Red-eyed female	61
White-eyed female	0
Red-eyed male	33
White-eyed male	6

1. Decide what percent of the total each group makes up. Then, draw a circle graph in the space to the right to represent the data in the table.

2. What question can the fruit fly data help answer?
- **a.** Do female fruit flies ever have white eyes?
- **b.** What makes a fruit fly have white eyes instead of normal, red eyes?
- **c.** Why are there more red-eyed male fruit flies than white-eyed male fruit flies?
- **d.** Will the same number of male and female offspring from this cross have white eyes?

3. What can you conclude from the fruit fly data?
- **a.** Female fruit flies can never have white eyes.
- **b.** About $\frac{1}{20}$ of the offspring from this cross have white eyes.
- **c.** About $\frac{1}{3}$ of all male fruit flies have white eyes.
- **d.** It is better for fruit flies to have red eyes than to have white eyes.

4. Why is it important to include in the data whether the fly is male or female?
- **a.** to find out whether the white-eyed mutation has anything to do with sex
- **b.** to make sure that there were an equal number of male and female flies
- **c.** to give the students practice at telling male and female fruit flies apart
- **d.** to give the students another characteristic of the fruit flies to look for

Name Date Class

Activity **13**

NCTM Standard:

Data Analysis and Probability

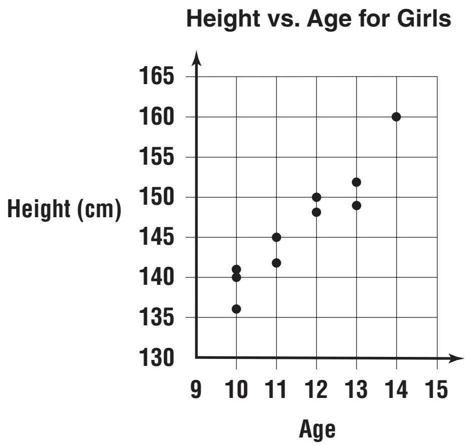
Review

What you should know about . . .	Find it by . . .	Example
Mean	Adding together all of the data, then dividing by the total number of data	If the number of cricket chirps heard by someone in 60-s units of time on 7 days is listed in order as 50 52 52 54 56 59 63, the mean is 55.
Median	Identifying the middle number when all of the data are written in order from least to greatest	The median is 54.
Mode	Identifying the most common item of data in the data set	The mode is 52.
Interquartile range	Identifying the middle part of the data by dividing the data before and after the median into two halves each	The lower quartile is 52. The upper quartile is 59.

Example

Mr. Jacoby's class collected data on ten girls at their school. They recorded the girls' ages and their heights. The results are shown in the scatter plot in **Figure 1**.

Figure 1



How are the ages of the girls related to the heights of the girls?

Solution Look at the scatter plot. A best-fit line is a straight line that passes close to most of the data points. Draw a best-fit line through the data. The line will go up from the left to the right, showing a positive relationship. The older the girls are, the taller they are.

Name ______________________ Date ____________ Class ____________

Activity 13 (continued)

Practice

Large numbers of gray wolves lived in Yellowstone National Park at one time. But in the early 1900s, gray wolves disappeared from Yellowstone completely. In the mid-1990s, 31 wolves were reintroduced to the park. By the beginning of 2000, an estimated 16 packs of wolves lived in the park. **Table 1** shows the number of wolves in each pack.

Table 1

Yellowstone Gray Wolf Pack Size
January 2000

Wolf pack name	Total number of observed wolves
Druid Peak	8
Rose Creek	16
Leopold	11
Crystal Creek	13
Chief Joseph	8
Nez Percé	13
Soda Butte	3
Sheep Mountain	9
Teton	6
Gros Ventre	5
Sunlight Basin	9
Taylor	2
North Fork Creek	2
North Greater Yellowstone Area	4
Washakie	2
Lone Wolves	7

Source: U.S. Fish and Wildlife Service

1. What is the mean number of wolves in a pack?
a. 7.3 **c.** 8.1
b. 7.4 **d.** 9.2

2. What is the mode of the wolf pack data?
a. 2.0 **c.** 8.0
b. 7.5 **d.** 9.0

3. What is the median of the wolf pack data?
a. 7.0 **c.** 8.0
b. 7.5 **d.** 8.5

4. What are the lower and upper quartiles of the wolf pack data?
a. 2, 8 **c.** 4, 11
b. 3, 9 **d.** 3.5, 10

5. What is the interquartile range of the wolf pack data?
a. 4.0 **c.** 6.5
b. 6.0 **d.** 7.0

6. Draw a box-and-whisker plot for the wolf pack data in the space provided below.

Name Date Class

Activity 14

NCTM Standard:

Data Analysis and Probability

Review

What you should know about probability:		Example
Probablility can help you **make and test hypotheses** and help you know whether the results in an experiment are likely to be correct.		What is the likelihood that a feather dropped from 5 m will land in a circle on the floor?
Mutually exclusive events		Either the feather will land in the circle or it won't.
Mutually inclusive events		If the circle on the floor intersects a square, the feather could land in both the circle and the square.
The **probability** of an event always **ranges from 0 to 1**	Probability of 0	If there is no circle on the floor, the feather cannot land in a circle.
	Probability of 1	The feather will land either in the circle or outside the circle. These are **complementary events**—Two possible events exist whose probabilities add up to 1.

Example

Karen purchased 10 rose bushes at the store. When they bloom, 6 of them will have red flowers and 4 of them will have yellow flowers. When Karen got home, she realized that the bushes were not labeled. She did not know which would have red blooms and which would have yellow blooms. What is the probability that the first bush that blooms will have yellow flowers? What is the probability that it will have red flowers? Are these two events complementary? Are they mutually exclusive?

Solution There are 10 rose bushes total. Although 4 of them are yellow, Karen does not know which ones. The probability of the first plant having yellow flowers is 4 out of 10. This can be written as the fraction $\frac{4}{10}$, which is equal to 0.40. The probability that the bush will have red flowers is $\frac{6}{10} = \frac{3}{5} = 0.60$. The probabilities of both events add up to 1, so they are complementary. They are also mutually exclusive because they cannot happen at the same time. The first bush to bloom cannot have both red and yellow flowers.

Activity 14 (continued)

Practice

Alec's class collected snails from a stream. They looked at the snails under microscopes to see what sex they were and whether they had parasites. They had looked at a total of 200 snails when **Table 1** was completed.

Table 1

	Parasites	No parasites
Male	51	49
Female	25	75

1. What is the probability that the next snail they look at will be male?

a. $\frac{1}{2}$ **c.** $\frac{1}{8}$

b. $\frac{1}{4}$ **d.** $\frac{1}{5}$

2. What is the probability that the next snail they look at will be male and have parasites?

a. $\frac{1}{2}$ **c.** $\frac{1}{2}$

b. $\frac{1}{4}$ **d.** $\frac{1}{16}$

Erika's physics class made rockets. They launched 18 of the 20 rockets. Of the rockets launched, 12 launched without a problem, but 6 of them did not work.

3. Draw a tree diagram to show how you would calculate the probability that 1 of the last 2 rockets will launch without a problem and the other will not work. What is the probability that this will happen?

Activity 15

NCTM Standard:

Problem Solving

Review

Sometimes a math problem tells you how to solve it—"Multiply the following." Other times, however, you must determine how to solve it. There are many ways to solve a problem. Many problem-solvers follow a four-step plan:

- First, **explore** the information in the problem. At this point, you should jot down essential information from the problem.
- Second, **plan** how you will solve it based on the information in the problem. Choose from the following strategies: make a table, make a list, act it out, work backward, use a Venn diagram, look for reasonable answers, look for a pattern, solve a simpler problem, make a model, draw a graph or diagram, guess and check, write an equation, or eliminate possibilities. Sometimes a combination of two or three strategies works best.
- Third, **solve** the problem.
- Finally, **examine** the answer. Is it reasonable? Does it come close to your estimated answer? If not, choose a different strategy and approach the problem from a different angle. Compare answers.

Example

Chemists use all kinds of chemical solutions. Normally, chemistry laboratories purchase the solutions that they use in concentrated form. Then, they dilute them with water. If a chemist needs a 10 percent solution of acetic acid (10 parts acid and 90 parts water), how much acid should he or she add to 0.5 L of water to make a 10 percent solution?

Solution This problem can be solved by using a proportion. The proportion of acid to water in the solution should be 10 to 90, or 1 to 9. Write the proportion $\frac{1}{9} = \frac{x}{0.5}$ L, where x = the amount of acid. Solving the proportion for x gives $1 \times 0.5 = 0.5$. The chemist should add $0.5 \div 9 = 0.06$ L of acid or 6 mL of acid to the 0.5 L of water.

Figure 1

Activity 15 (continued)

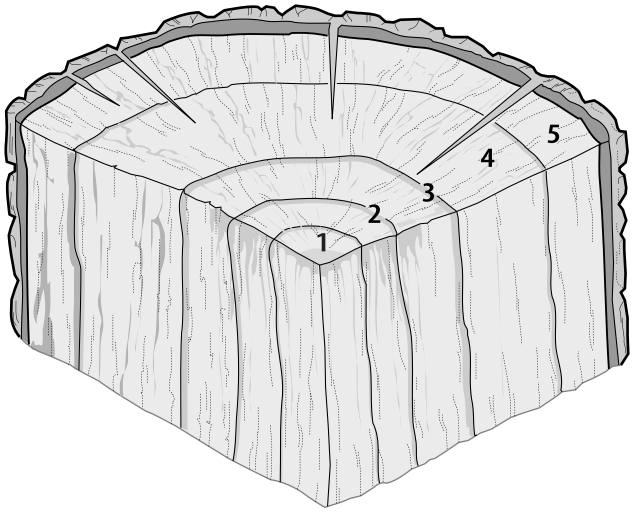
Practice

If you look at a tree stump, you will see that it has rings—one ring for each year of its life. Scientists study these rings to learn about climate and to date past climatic events. If a tree has a thick ring, it probably rained a lot that year. If a tree has a thinner ring, there may have been a drought that year.

1. The diameter of this tree stump is 29 cm. What is the approximate circumference of the stump? Use 3.14 for π.

a. 660 cm
b. 46 cm
c. 91 cm
d. 182 cm

Figure 2



2. If the radius from the center of the stump to the outer edge of ring 2 is 2 cm, and the radius from the center of the stump to the outer edge of ring 4 is 12 cm, about how much bigger around was the tree when it was 4 years old than when it was 2 years old?

a. 62.8 cm **c.** 78.52 cm
b. 87.92 cm **d.** 31.4 cm

3. Assume that the diagram of the tree stump is **NOT** drawn to scale. Based on other information, which of the following is most likely to be true?

a. There was a drought in the fifth year.
b. It rained more during the fifth year than during all of the previous years combined.
c. It rained less during the fifth year than during all of the previous years combined.
d. It rained the most during the first year.

4. Automobiles typically use gasoline for energy. The Environmental Protection Agency publishes annual reports comparing the fuel efficiency of all commercial vehicles. It uses miles-per-gallon ratings to tell how far an automobile will travel on one gallon of gasoline. If an automobile has a rating of 25 to 28 miles per gallon and its gas tank holds 15.5 gallons, what is the maximum distance that it can travel on a full tank before it needs to be refueled?

a. 334 miles **c.** 387 miles
b. 443 miles **d.** 434 miles

Name Date Class

Activity 16

NCTM Standard:

Problem Solving

Review

Some problems can have more than one answer, depending upon how you solve them. In such cases, you can analyze the solutions that you get and compare them with ones that your classmates come up with. Do other solutions exist?

Example

What equation can you write to describe the data in the table? Use the equation to predict how many fish pellets 50 fish would need per week.

Table 1

Number of fish	Number of fish pellets per week
10	5
20	10
30	15
40	20

Solution At least two equations will work to describe the pattern.

1. As the number of fish increases by 10, the number of fish pellets increases by 5. Therefore, if F = the number of fish, P = the number of fish pellets per week, and x = the number of 10-fish increases, then

$$P_{(x+1)} = P_x + 5.$$

Put in another way, a series describing this pattern would be

$$P + (P + 5) + [(P + 5) + 5] + \ldots$$

2. The number of fish pellets also equals half the number of fish. Therefore,

$$P_x = \frac{F_x}{2}.$$

According to both equations, 50 fish would need 25 fish pellets per week.

Practice

1. Shaneece needs to get 10 mL of sulfuric acid from the teacher for her lab partner and herself for an experiment. Can she use a beaker that has a 1.5-cm diameter and is 7 cm high in order to hold the sulfuric acid? Explain.

2. Mr. Culler's biology class is dissecting frogs. The average length of the frogs with their arms and legs stretched out is 35 cm. The class has four different-sized square dissection trays. Which size of dissection tray will be the best fit for the frogs?
   a. 2,500 cm^2
   b. 400 cm^2
   c. 4,900 cm^2
   d. 1,600 cm^2

3. Glacier lilies are wildflowers that grow in the mountains of Utah, Montana, Idaho, and Washington. One summer, there were 40 glacier lilies in Megan's backyard. The next summer Megan reported to her parents that the number of lilies had increased by 5 percent. How many new lilies were there?
   a. 1
   b. 2
   c. 3
   d. 4

4. Ornithologists study birds. One ornithologist counted the number of visits by hummingbirds at three different locations, A, B, and C, each with different flowers. Over three weeks at location A, he counted 28 hummingbirds. Over 2 weeks at location B, he counted 54 hummingbirds. Over 4 weeks at location C, he counted 72 hummingbirds. Which location did the hummingbirds prefer? Explain how you arrived at your answer.

5. Pravat and Nick placed 100 mL of 25°C water in direct sunlight. Then, they measured the temperature of the water every minute. If the temperature increased by 0.5°C per min, what was the temperature of the water after 7 min?
   a. 3.5°C
   b. 12.5°C
   c. 25.5°C
   d. 28.5°C

Name Date Class

Activity **17**

NCTM Standard:

Reasoning and Proof

Review

The rules and techniques that you study in math class developed as people observed the world around them. They examined patterns and noted how numbers related to natural phenomena. This way of thinking is called *inductive reasoning:*

1. Observe a pattern.
2. Form a hyphothesis.
3. Make a rule to describe the pattern.
4. Test the rule.

Example

Table 1 shows the number of bacteria colonies growing in a petri dish every day for seven days. Form a hypothesis that describes the data. How many bacteria colonies should there be on the tenth day?

Table 1

Day	1	2	3	4	5	6	7
Number of bacteria colonies	3	6	12	24	48	96	192

Solution Study the information in the table to try to find a pattern. The original number of bacteria colonies was 3. The next day, the number had doubled to 6. The following day it had doubled again to 12. In fact, each day, the number of bacteria colonies doubles. A good hypothesis would be, "The number of bacteria colonies each day is twice as many as the day before." On the tenth day there should be 1,536 bacteria colonies.

Practice

Jonathan had to create a hypothetical animal for a biology class. He needed to be very specific about which characteristics his animal would have. He chose to create a snake and gave it black and white bands. He then created a data table to show how many bands the snake should have depending on its length. This information is given in **Table 2.**

Table 2

Length of snake (cm)	Number of bands
20	10
30	13
40	17
50	22

Activity 17 (continued)

1. Write a rule to describe the pattern of the data in **Table 2**.

2. Write an equation that you can use to test your rule.

3. Use your rule to find out how many bands a snake that is 70 cm long will have. Show your work.

4. If this were an actual snake, how could you test your rule?

Name Date Class

Activity **18**

NCTM Standard:

Reasoning and Proof

Review

One way of thinking is called *deductive reasoning*. This type of reasoning starts with a hypothesis. Then, you must prove or disprove the hypothesis.

Disprove a mathematical statement by finding one example for which the statement is false.	"A shape that is not square cannot have four 90° angles." Disproved because all rectangles have four 90° angles.
Prove a mathematical statement true by using a logical argument, such as a formal proof.	"All right angles are congruent." **1.** Given: $\angle 1$ and $\angle 2$ are right angles. **2.** $m\angle 1 = 90°$, $m\angle 2 = 90°$, by the definition of right angles. **3.** $m\angle 1 = m\angle 2$, by the substitution property. **4.** $\angle 1 \cong \angle 2$

Example

After studying several birds, Ayisha observed that they all lay eggs. She then came up with the following rule: "Any animal that lays eggs is a bird." Disprove her rule.

Solution To disprove a rule, find one case for which the rule is false. In this case, find an animal that lays eggs that is not a bird. There are many examples. All fish and most reptiles lay eggs. Some mammals even lay eggs, namely the duck-billed platypus and the spiny anteater. This fact disproves Ayisha's rule.

Practice

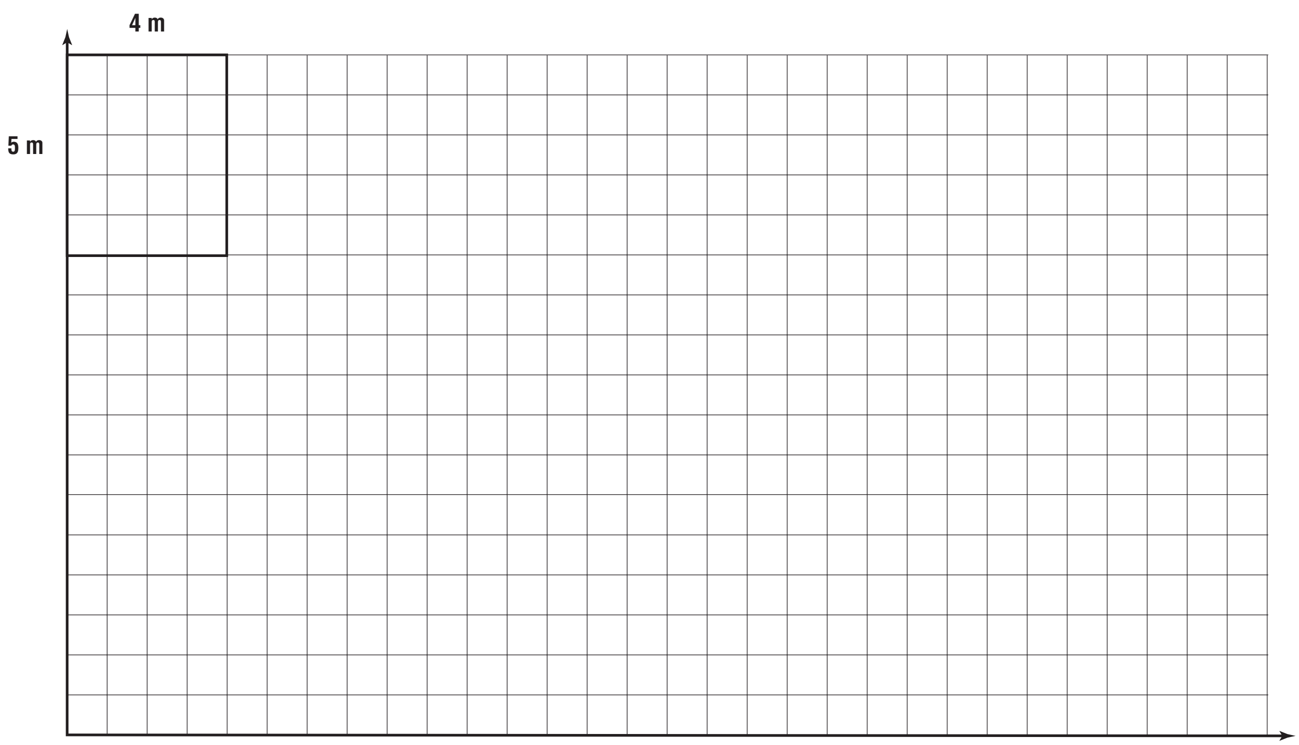
1. "Ice will melt into water if it is warmer than 0°C." Create a new rule based on this rule.

Activity 18 (continued)

2. Give an example that proves or disproves your new rule about ice.

3. Sue observed that all mammals breathe air. She then thought that if all mammals breathe air, all must live on land. Give an example that proves Sue's rule is not true.

4. Ravi is planting a vegetable garden. He wants the garden to be 20 m^2 and rectangular. He thinks that the only way to do this is to make a rectangle that is 4 m wide and 5 m long like the one shown. Use the gridlines to help you prove Ravi wrong.



Name Date Class

Activity 19

NCTM Standard

Communication

Review

Use communication as a tool for solving a math problem:

- Organize your thoughts about that problem into sentences or a picture.
- Listen to the thoughts of other students.
- Explain your solution to the class.
- Compare your solution with other students' solutions.
- Find the flaws in the explanations of others.

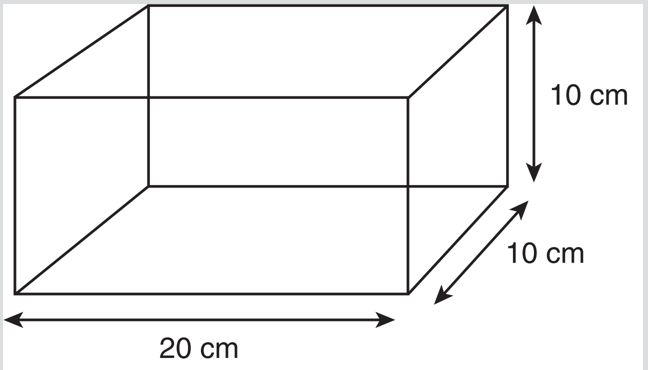
Example

Look at the following problem. Explain to your brother or a friend how to solve the problem. Include a drawing to help him or her understand the solution.

Ashon wanted to get some new fish for his fish tank. The sales clerk at the pet store told him that he should estimate that each fish needs about 100 cm^3 of space. Ashon's fish tank is 20 cm × 10 cm by 10 cm. He already has 5 fish in the tank. How many more fish can he add?

Solution Draw a picture of the fish tank so that it is easy to see its size. Then, find the volume of the fish tank.

Figure 1



$$\text{Volume} = \text{length} \times \text{width} \times \text{height}.$$

So, the volume of the fish tank = 20 × 10 × 10 = 2,000 cm^3.

Now, divide the volume of the fish tank by the amount of space each fish needs:

$$2{,}000 \div 100 = 20.$$

Ashon's fish tank can hold 20 fish. If Ashon already has 5 fish, he still has room for 15 more, because 20 − 5 = 15.

Activity 19 (continued)

Practice

The students in a science class performed an experiment to see how temperature and pressure are related. They started with two full plastic bottles of carbonated soft drink. They poured half of one bottle out, and labeled it bottle #1. They labeled the other bottle #2. Then they put one-holed rubber stoppers in the bottles. They inserted a thermometer into the hole of each rubber stopper and measured the temperature in each of the bottles. Each bottle started out at room temperature, 23°C. Then, they shook bottle #1 four times and again recorded the temperatures of the two bottles. They continued to measure the temperatures of the bottles after every four shakes until they had shaken bottle #1 20 times. They did not shake bottle #2. They recorded their data in **Table 1**.

Table 1

Total number of shakes of bottle #1	Temperature of bottle #1 (°C)	Temperature of bottle #2 (°C)
0	23	23
4	24	23
8	25	23
12	26	23
16	27	23
20	28	23

1. Make a graph of the data on a separate sheet of paper. Then, using the graph, explain how temperature and pressure are related in this experiment.

2. The class wrote an equation to figure out the temperature of the soda in bottle #1 after x number of shakes (T = the temperature):

$$23°\text{C} \times \frac{1}{4}x = T.$$

Explain how the class came up with the equation.

Activity 20

NCTM Standard:

Communication

Review

Communication is a two-way street. Just as you need to make yourself understood by others, you need to be able to process and understand the thoughts of the people you are working with. You often can find the solution to a problem by discussing it with others. As you explain to one another how you tackled a problem, you can catch one another's mistakes and fix them, thus arriving at the correct answer.

Example

Pek Lee just read an article on high cholesterol. In the United States, a significant number of men and women have high levels of cholesterol in their blood. Health experts believe that too much blood cholesterol is unhealthy, especially for the heart.

Pek Lee recorded information from the article into **Table 1**. She is explaining to her classmates how she used the table to determine about how many adults in the United States have high cholesterol.

Table 1

Percent of U.S. Population with High Blood Cholesterol (1988–1994)

Sex	Age (years)					
	20–34	35–44	45–54	55–64	65–74	75+
Female	7.3	12.3	26.7	40.9	41.3	38.2
Male	8.2	19.4	26.6	28.0	21.9	20.4

Source: Centers for Disease Control and Prevention, National Center for Health Statistics, Division of Health Examination Statistics

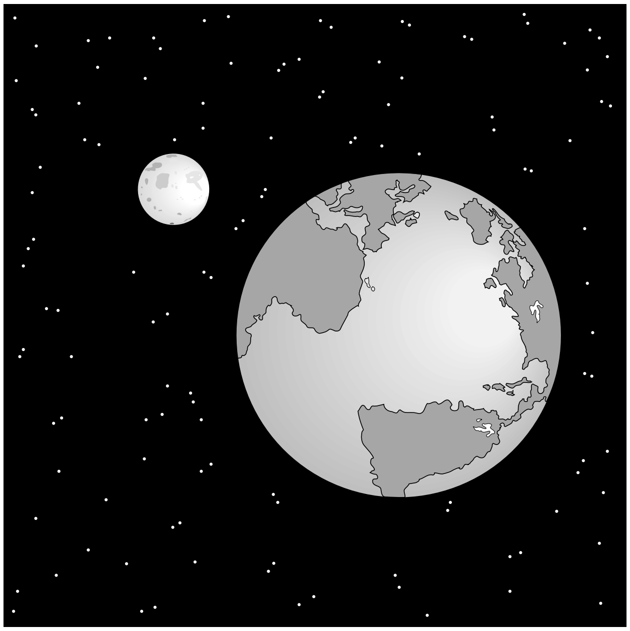
Pek Lee's explanation: "Add up all of the percents in the table and divide by 12. About 24 percent of adults in the United States have high cholesterol."

How is Pek Lee's explanation incomplete? What more should she have included? Did she make any mistakes?

Solution She does not explain why she adds up all of the percents or why she then divided them by 12. She should explain that she is finding an average percent for all adults by adding all of the percents and dividing the sum by 12, which is the total number of percents. Pek Lee did not take into consideration that there may be more people in one age group than in another, or that high cholesterol may be more of a problem for some age groups than others. In fact, the study's authors concluded that about 19 percent of adults in the United States have high cholesterol.

Activity 20 (continued)

Practice



Justin's class was asked to make a scale model of Earth and the Moon and the distance between them. His teacher told the class that the diameter of the Moon is nearly 3,480 km, the diameter of Earth is about 12,700 km, and that Earth is about 384,000 km from the moon. Then she gave each group a small marble to represent the Moon in the scale model. It had a diameter of 2 cm. She then asked them to determine how big the model of Earth should be and how far away from the Moon it should be. Justin wrote down what his group did to figure out the answer.

Justin's explanation:

"The first thing we did was figure out how much bigger Earth is than the Moon. It is 3.6 times bigger. That means that the model of Earth should be 3.6 times bigger, so it should be 7.2 cm. To figure out how far apart the models of Earth and the Moon should be, we divided the actual distance by 3.6. The models of Earth and the Moon should be 106,000 cm apart."

1. What is unclear about how Justin's group determined the size of the Earth for the model?

2. What is unclear about how his group determined the distance between Earth and the Moon for the model?

3. Which of their calculations is incorrect?

4. Is it possible to make a model like the one Justin describes? Explain.

Name Date Class

Activity 21

NCTM Standard:

Communication

Review

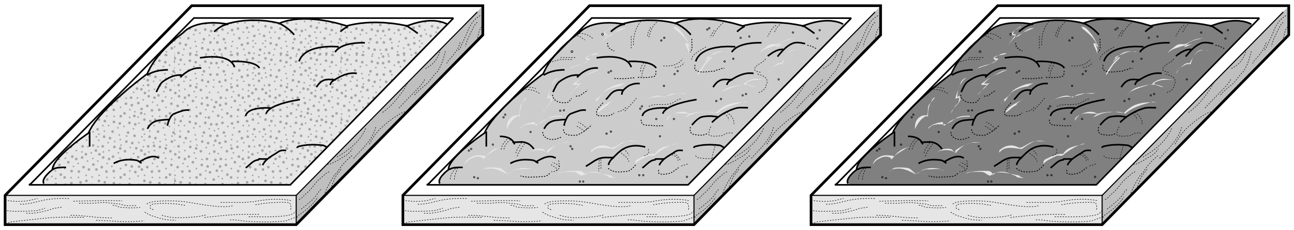
Sometimes it may seem as though math is nothing but strange symbols. You may have wondered why mathematicians don't just say things the simple way. Well, they do. By using mathematical symbols as a sort of shorthand, mathematicians can say quickly and clearly what might take a lot longer to say in everyday language. If you don't believe this, find something that looks complicated in your math book. Try to communicate what everything means without using any math, including the words *plus, minus,* or *equals*. It might be harder than you think!

Example

Study the following problem. Express the problem mathematically.

Erin has just completed an Earth science experiment in order to see how much water different types of soil can soak up. In her experiment, sand absorbed one-half as much water as potting soil, and twice as much water as clay. Use the variable P for the amount of water the potting soil absorbed, S for the amount of water the sand soaked up, and C for the amount of water the clay soaked up. In mathematical language, express the amount of water that sand absorbed compared to the other two soils.

Figure 1



Solution Read the problem carefully. It says that sand soaked up half as much water as potting soil. This can be written as $S = \frac{1}{2}P$. It also says that sand soaked up twice as much water as clay. This can be written as $S = 2C$. You can express all of the relationships at once by writing that $\frac{1}{2}P = S = 2C$.

Activity 21 (continued)

Practice

1. To find the volume of a rock, Soto put it in a measuring cup that was filled with water. The water level rose when he added the rock. Which mathematical sentence can he use to find the volume of the rock?
   - **a.** Volume of rock − volume of water with rock = volume of water
   - **b.** Volume of water − volume of rock = volume of water with rock
   - **c.** Volume of rock + volume of water with rock = volume of water without rock
   - **d.** Volume of water with rock − volume of water without rock = volume of rock

2. The playground at Lincoln Elementary has 5 different types of surfaces. One-third of it is covered with cut-up tires. One-quarter is gravel. One-twelfth of it is asphalt. One-sixth of it is sand and the final sixth is grass. In **Figure 2** draw a circle graph that communicates the amount of playground each type of surface covers.

Figure 2

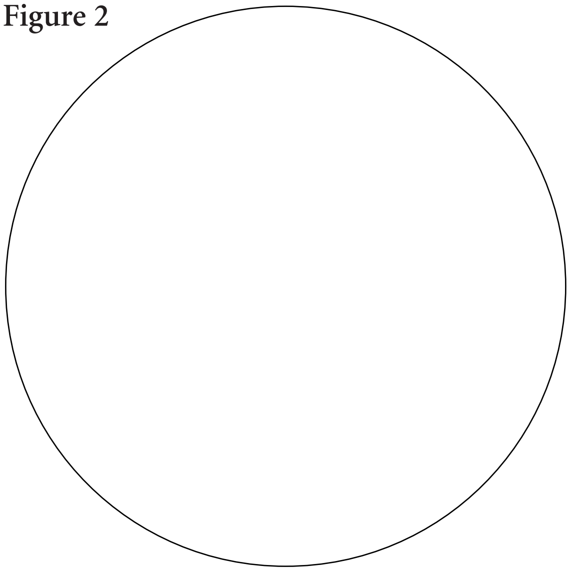
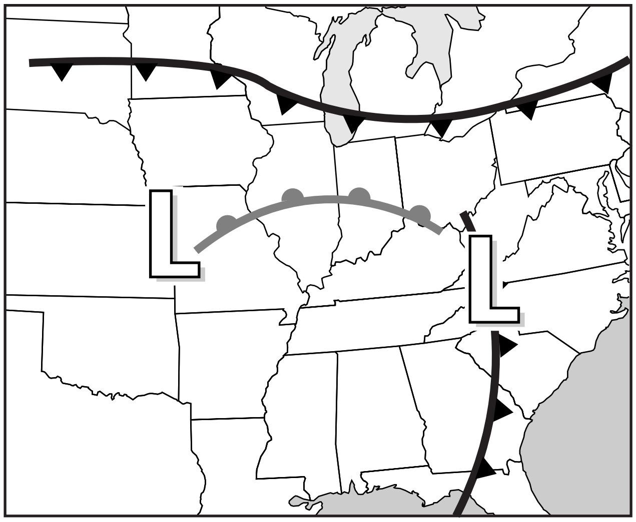


Figure 3



3. On weather maps such as the one in **Figure 3**, weather forecasters use a solid blue line with triangles on it to show a cold front. The triangles point in the direction that the cold front is moving. If the triangles are equilateral and the sum of the sides of a triangle equals its area, what is the height of the triangle? Express this problem mathematically. (s = side, h = height)
   - **a.** $3s = \frac{1}{2}sh$
   - **b.** $\frac{1}{2}s = 3h$
   - **c.** $3h = \frac{1}{2}sh$
   - **d.** $\frac{1}{2}h = 3sh$

4. Male Bengal tigers can leap more than three times farther than their body length, which averages 2.9 m. If a male Bengal tiger is l m long, what is the minimum distance (d) that it is capable of jumping? Express this problem mathematically.
   - **a.** $3d = l$
   - **b.** $3l = d$
   - **c.** $\frac{1}{3}l = d$
   - **d.** $d = l$

Activity 22

NCTM Standard:

Connections

Review

Sometimes in school it can seem as though everything you learn is separate and self-contained. What does algebra have to do with geometry or, certainly, social studies class? Hard as it may seem, however, many of the things you learn are connected and, in fact, make more sense when you can see the connections. Instead of plunging immediately into a new problem, for example, take a moment to think about similar problems you may have seen before. Maybe this problem uses ratios and proportions. What is it about those other problems that might help you with this one? Suddenly this new problem won't seem so unfamiliar to you.

Example

Solve the following problem.

Jeffrey has a rectangular solar cell that is 4 cm long and 3 cm wide. The solar cell generates an electrical current of 0.03 amps when it is placed in full sunlight. Assume that the area of the solar cell and the amount of current it produces are directly proportional. If Jeffrey has another rectangular solar cell that is 9 cm long and 2 cm wide, what will its current be in full sunlight?

Solution Read the problem carefully because its solution is based on connections among several mathematical ideas—area, ratios, proportions, and decimals. Find the area of the first solar cell. Remember that the area of a rectangle is equal to length times width, or

$$A = l \times w$$
$$A = 4 \text{ cm} \times 3 \text{ cm}$$
$$A = 12 \text{ cm}^2.$$

Then, find the area of the second solar cell:

$$A = 9 \text{ cm} \times 2 \text{ cm}$$
$$A = 18 \text{ cm}^2.$$

To find the current generated by the second solar cell, set up a proportion using the ratio of area to current for each cell. You then can solve for the unknown variable in the proportion, *I*, which represents the current of the second solar cell. To solve a proportion for an unknown, cross multiply, then solve for the variable.

$$\frac{12 \text{ cm}^2}{0.03} = \frac{18 \text{ cm}^2}{I} \text{ or } \frac{12}{0.03} = \frac{18}{I}$$
$$12\,I = 0.54$$
$$I = 0.045.$$

The current generated by the second solar cell is 0.045 A.

Activity 22 (continued)

Practice

Use the following information to answer questions 1–4.

Tanisha made a solution of sodium chloride (NaCl) and water by adding 28 g of NaCl to 1 L of water. The concentration of this solution is expressed in grams per liter (g/L). In this case, since she added 28 g of NaCl to 1 L of water, the concentration of the solution is 28 g/L.

1. Tanisha wants to add water to the flask in order to create a second solution that is only 25 percent as concentrated as the original formula. What is the first step she must take in order to figure out how much water to add?
- a. Multiply the concentration of the original solution by 25 percent in order to find the concentration of the new solution.
- b. Multiply the volume of the original by 25 percent.
- c. Add 25 percent to the volume of the original.
- d. Divide the concentration of the original solution by 25 percent in order to find the concentration of the new solution.

2. What will be the concentration of Tanisha's new solution?
- a. 112 g/L
- b. 28 g/L
- c. 14 g/L
- d. 7 g/L

3. Recall that Tanisha has 1 L of a solution that has a concentration of 28 g/L. How much water will she have to add to this solution in order to lower the concentration that you found in Exercise 2 above?
- **a.** 8 L
- **b.** 4 L
- **c.** 3 L
- **d.** 1 L

4. Suppose that Tanisha wants to make a solution that is 80 percent as concentrated as her original solution. How much water would she have to add to her 1 L of solution? What would be the new concentration?
- **a.** Add 1 L, the concentration is 26 g/L.
- **b.** Add 500 mL, the concentration is 25 g/L.
- **c.** Add 250 mL, the concentration is 22.4 g/L.
- **d.** Add 100 mL, the concentration is 20 g/L.

Activity 23

NCTM Standard:

Connections

Review

When you close your math book, do you feel like you're finished with math for the day? Do you ever use math in science class? In social studies class? If you asked everyone in your school how they would vote in an upcoming election if they were older, how would you analyze your results? You would need to use math to add up all of the responses, calculate percents of students voting for each candidate, and decide on a method in which to represent the data.

Example

Solve the following problem.

Erin and her lab partner Fillip used a microscope to count the number of bacteria that were growing in a nutrient-filled petri dish. They noticed that the number of bacteria doubled every 20 min as the bacteria cells divided. If there were 64 cells when Erin and Fillip first looked at the bacteria, how many cells should they expect to find in three hours?

Solution Bacteria cells reproduce by dividing. Therefore, any time a group of bacteria cells divides, twice as many bacteria result as there were before they divided. This type of growth is called exponential growth because it can be represented using exponents. Because each bacteria divides into 2, the expression 2^x can be used to calculate the number of bacteria cells that will exist after x divisions.

Erin and Fillip observed that their bacteria divided every 20 min. You need to know how many times the bacteria divide in an hour to solve the problem. Since 20 min is $\frac{1}{3}$ h, the bacteria will divide 3 times in one hour. Because Erin and Fillip are watching the bacteria for three hours, and the bacteria divide three times in each hour, the bacteria will divide 3×3, or 9, times. To find the number of bacteria after 9 divisions, starting with 64 bacteria, multiply 64 by 2^9:

$$64 \times 2^9 = 32{,}768.$$

After three hours, Erin and Fillip should expect to see 32,768 bacteria.

Practice

1. The golden-cheeked warbler (*Dendroica chrysoparia*) is a unique songbird. It breeds only in central Texas in Ashe juniper woodlands. In 1990, the golden-cheeked warbler joined the list of endangered species. As nearby cities have grown, developers have cut down many of the Ashe juniper trees in which the songbird makes its nests. In order to protect the golden-cheeked warbler and other birds, the Balcones Canyonlands National Wildlife Refuge was set up near Austin. Eventually, the refuge will cover about 18,600 hectares.

Saffron's science class visited the refuge in the spring. The students helped a wildlife biologist locate golden-cheeked warbler nests in 8 hectares of the refuge. The students and the biologist found two nests in the 8-hectare search area. According to the biologist, two adult warblers exist for each nest. If all of the land on the completed refuge is suitable habitat for golden-cheeked warblers, about how many adults of the species would the biologist expect to find on the whole refuge?

a. 4,650 **c.** 37,200
b. 9,300 **d.** 74,400

Use the following situation to answer questions 2–3.

Lee's science fair project is about conserving water. She wants to show how rainwater can be stored and used as an alternative source of water for irrigating lawns. For her demonstration, she built a plastic box that was 50 cm wide, 70 cm long, and 20 cm high. She left the box open on the top so it can collect rainwater. After each rainfall, she will measure the amount of water in the box.

2. After the first rainstorm, Lee ran outside to see how much water was in the box. She was disappointed to find that the wind during the storm had blown the box over, and it did not collect any rainwater. Her father's rain gauge showed that it had rained 4 cm. Explain how Lee can use geometry to figure out how much water would have been in the box. ***Hint***: *1cm³ of water = 1 mL of water.*

3. How much water would Lee have in the box after a 4-cm rain?

a. 140 L **c.** 14 L
b. 70 L **d.** 7 L

Name Date Class

Activity 24

NCTM Standard:

Representation

Review

Mathematical representations make it easy to show complex ideas quickly and efficiently:

When you read . . .	You should think of . . .
$y = mx + b$	a straight line
2×10^8	a large number
$2\pi r$	a circle
$(-3, 0)$, $(0, 3)$, $(3, 0)$, and $(0, -3)$ plotted and connected on a coordinate grid	a square

Other representations include physical models; drawings; charts, tables and graphs; symbols, including exponential and scientific notation; graphing calculators; and spreadsheets. Choosing an effective representation can help you communicate your ideas to others, solve a problem, or intrepret data.

Example

Solve the following problem.

Ronda wanted to include a poster in her presentation on the constellation Cassiopeia. She knew that anything that she drew on the poster would be two-dimensional, but that the relationship among the stars and other objects in the constellation was three-dimensional. She also knew that the stars in the constellation were of different sizes and magnitudes. How could she represent all of these aspects of Cassiopeia on a poster?

Solution Ronda decided to show the major stars in Cassiopeia in position over a traditional drawing of the mythical Ethiopian queen Cassiopeia sitting on her throne. This way, she thought that the class might remember better both the relative positions of the stars and the name and story behind the constellation. Ronda also decided to include a table on her poster with data on the major stars in Cassiopeia. She included the names and visual magnitudes of the stars, along with their average distances from Earth.

Table 1

Name	Alternate name	Visual magnitude	Average distance from Earth (light years)
Alpha Cas	Shedir or Shedar	2.23	140
Beta Cas	Caph	2.27	49
Gamma Cas	Cih	2.47	850
Delta Cas	Rucha or Ruchbah	2.68	88
Epsilon Cas	Segin	3.38	590

Name Date Class

Practice

Zack and several of his classmates have just finished lab on rock density. They want to find out what the results they got mean. First they organize the data into a table.

Figure 1

	A	B	C	D	E	F
1				Rock density (g/mL)		
2	Student	Rock type	Sample A	Sample B	Sample C	Average density
3	Adriana	Basalt	2.79	2.95	2.84	
4	Chase	Shale	2.54	2.51	2.57	
5	Habiba	Granite	2.67	2.59	2.72	
6	Nathan	Marble	3.01	2.86	2.66	
7	Tatyana	Gneiss	2.81	2.82	2.82	
8	Zack	Andesite	2.82	2.84	2.75	

1. They want to make a bar graph of the data. Tatyana then suggests that they average the density readings for each type of rock. That way, they can use one bar for each type of rock. She suggests using an electronic spreadsheet. Together, they put together the spreadsheet shown in **Figure 1**.

   Which of the following cells in the spreadsheet in the spreadsheet would be included in a formula to find the average density of shale?
   a. A4, B4, and C4
   b. A5, B5, and C5
   c. C4, D4, and E4
   d. F4

2. Which of the following formulas would calculate the average density of Gneiss?
   a. = (C7 + C8 + C9) ÷ 3
   b. = (C7 + D7 + E7) ÷ 3
   c. = (C7 + D7 + E7) ÷ 2
   d. = (F7) ÷ 3

3. Suppose that the group also wanted to find the average density of all of the rocks, based on the average density of each type of rock. Which of the following formulas would work?
   a. = (F3 + F4 + F5 + F6 + F7 + F8) ÷ 6
   b. = (F3 + F4 + F5 + F6 + F7 + F8) ÷ 3
   c. = (A9 + B9 + C9 + D9 + E9) ÷ 5
   d. = (A9 + B9 + C9 + D9 + E9 + F9) ÷ 6

4. Draw a bar graph showing the average densities of rock found by Zack and his group.

Name Date Class

Activity 25

NCTM Standard:

Representation

Review

When you have a table of data, you should be able to show those data on a graph. Many types of graphs exist, however—circle graphs, line graphs, bar graphs, scatter plots, and so forth. How do you decide on the best type of graph for the data? You need to choose ways to represent your data that clearly communicate the behavior of the data. Do they have a linear relationship, or are they geometric, or random? Sometimes you can predict further data by extending the graph, or by finding an equation to represent the relationships among the data.

Example

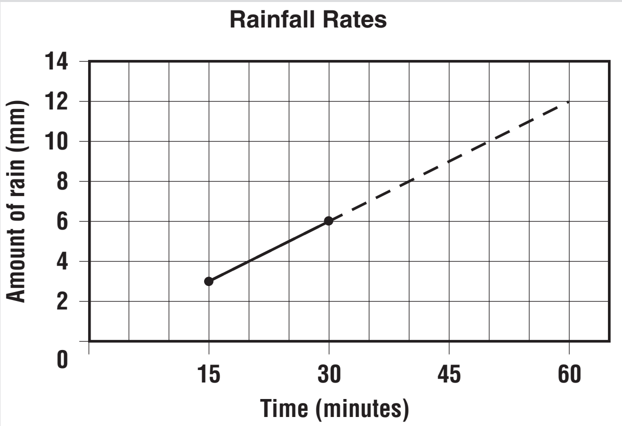
Madison and Mike are measuring rainfall rates for an Earth science project. To find the rate at which rain is falling, they measure the amount of water in a rain gauge every 15 min while it rains. **Table 1** shows their measurements after 30 min have passed.

Table 1

Time elapsed (min)	15	30	45	60
Amount of rain in gauge (mm)	3	6		

Assuming that the rainfall rate is constant, how much rain will have fallen in 45 min? In one hour?

Solution Graph the rainfall amounts. Because the rate is constant, the graph is linear, or in a straight line. Extend the graph in a straight line to find out that it will have rained 9 mm in 45 min and 12 mm in 60 min. Because the graph is linear, a linear equation would describe the data best, that is $y = mx + b$, where m is the slope of the line and b is the y-intercept. Therefore, $y = \frac{1}{5}x$.



Practice

During the summer, the level of Valley Reservoir usually drops because more water is being released from the reservoir than is flowing into it from the river. **Table 2** shows the weekly water level.

Table 2

Date	8-1	8-8	8-15	8-22	8-29	9-5
Reservoir level (m)	27.5	27.0	26.5			

1. Use the information in **Table 2** to make a graph. Be sure to include the dates that do not have measurements.

2. Assuming that the reservoir level keeps dropping as it did during the first three weeks of August, how can you predict the reservoir level on August 29 and September 5?

3. What will the reservoir level be on September 5?
   - **a.** 26.0 m
   - **b.** 25.5 m
   - **c.** 25.0 m
   - **d.** 24.4 m

4. What will the reservoir level be on August 29?
   - **a.** 27.0 m
   - **b.** 26.5 m
   - **c.** 26.0 m
   - **d.** 25.5 m

Activity 26

NCTM Standard:

Representation

Review

Many of the things that you investigate in the world around you can be recorded and represented using data tables and graphs.

Example: Suppose you took a survey in your school of the foods students eat for breakfast. You could make a graph of the results.

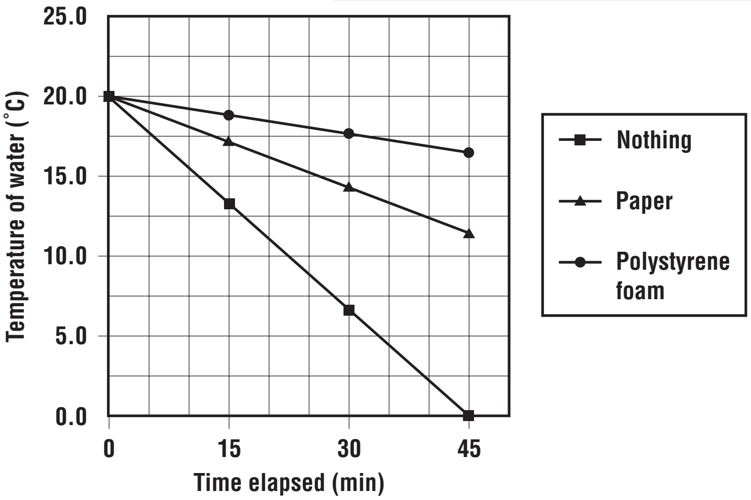
1. Using a frequency table, **tally** how many students prefer each type of food mentioned, such as cereal, eggs and bacon, bagels, toast, or pancakes.
2. **Determine** what percent of the total foods each food represents.
3. **Graph** the percents on a bar graph.
4. **Compare** the data. For example, if the bar for bagels is taller than the bar for pancakes, you can see immediately that there are more students who prefer bagels than students who prefer pancakes.

Example

Sarah and Kaleb are doing an experiment on the effectiveness of different types of insulation for their science fair experiment. They measured the rate at which the temperature of 100 mL of water in an ordinary glass drops when placed in a freezer at 0°C. They then insulated the glass with different materials and compared the rate at which the temperature dropped and recorded the data in **Table 1**. However, their science teacher said that the data would be much more effective if they were shown in a graph. What is the best kind of graph for these data? What would it look like?

Table 1

Time elapsed (min)	Temperature of water (°C) in a glass that is insulated with:		
	Nothing	Paper	Polystyrene foam
0	20.0	20.0	20.0
15	13.3	17.2	18.9
30	6.7	14.4	17.8
45	0.0	11.7	16.7



Solution The best kind of graph for a change in an amount over time is a line graph. The lines on the graph make it easy to compare the different values. When the insulation experiment data are graphed, it is easy to see that polystyrene foam is a much better insulator than paper or wood.

Activity 26 (continued)

Practice

Mr. Allegro's social studies class decided to survey the school to see what types of items students' families recycled on a regular basis. The students in the class organized the responses into **Table 2.** Assume that only one student per family responded and that the total number of students in the school was 792.

Table 2

Type of recyclable item	Number of families who recycle item regularly
Paper and paperboard	314
Compost	78
Metals	211
Glass	479
Plastics	117
All other	53

Table 3

Type of recyclable item	Percent of people in the United States who recycle item regularly
Paper and paperboard	58.0
Compost	17.1
Metals	11.0
Glass	5.5
Plastics	1.8
All other	6.6

1. The students in the class decided that they wanted to compare their data with national data to see whether the families in their school were typical. **Table 3** shows the national data. What did the students need to do first in order to compare the data?

a. Average the responses for their school.
b. Total the responses for their school.
c. Calculate what percent of families recycled each item.
d. Calculate what percent each item represented for the total number of recycled items.

2. After finishing the step described in Exercise 1, the students graphed the school and national data on one graph. Which of the following types of graphs would compare the data best?

a. circle graph
b. scatter plot
c. box-and-whisker plot
d. bar graph

3. On a separate sheet of paper, draw the graph that the students should have drawn.

4. Based on the graph, compared to the national figures, the families of students in the school ______.

a. do a better job recycling glass
b. do a better job recycling compost
c. don't recycle plastics as much
d. all of the above